DAR model notes – 2.0

**The DAR(1)-MA(1) model**

**Functional Form**

General form,

Can be rewritten as:

**Distribution**

We have derived that is conditionally distributed as,

Assuming that , the conditional distribution of is then,

and

As depends on both and , the model violates the Markov-chain assumption assumptin I.3.1 (i) in ARCH Part 1. However, we can reformulate the model on companion form as,

Using this formulation, conditional on the past values , depends only on as shown,

However, violates the assumptin I.3.1 (ii) in ARCH Part 1, as it is singular. This can be shown by using the formula for conditional densities , yielding,

As shown before, the first term is is continous Gaussian density. But, , since is fixed (already in the information set). Therefore, is no longer a random variable, and the conditional density function becomes a Dirac delta function at , which is not continues, violating the assumption I.3.1 (ii).

The problem of singularity can be solved by instead conditioning on ,

Since is a continious Gaussian density, it follows that is also. It holds that the product of two continous Gaussian densities is also a continous Gaussian density, therefore satisfies assumption I.3.1 (ii).

**Drift Criterion, 2.0:**

We consider the drift function

We start with finding

**QMLE:**

**General form DAR(1)-MA(1)-X,**

Can be rewritten as:

First we need conditional mean and variance:

**Conditional Mean:**

**Conditional Variance:**

The conditional variance can be decomposed as,

**Distribution**

We have derived that is conditionally distributed as,

Assuming that , the conditional distribution of is then,

With and

**Likelihood function:**

Ignoring constants:

**Drift Criterion, 3.0:**

**General form DAR(1)-MA(1)-X,**

We consider the drift function

Where

We start with finding

Can be rewritten as: